B. Math. III Topology Semestral Examination 2013

Attempt all questions. You may do your rough work on remaining pages or extra sheets (none of which will be graded). You may use books and notes.

1. *Write "True" or "False" for each of the parts (a) through (f) below. Each correct answer merits 4 marks, and each wrong answer gets 0 marks. In this first question, DO NOT furnish any arguments or reasons because these will not earn any credit.*

   (a): Let $K = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$, and $X = \mathbb{R}_K$ (the real line with the K-topology). Then K' (= the set of limit points of K in X) is empty. (True/False)

   (b): If A is an open subspace of a Lindelof space X, then A is also Lindelof. (True/False)

   (c): Let A and B be subspaces of a topological space X. Then $\overline{A} \setminus \overline{B} = \overline{A \setminus B}$. (True/False)

   (d): Let A be a subspace of S_Ω which is metrisable. Then A has finite cardinality. (True/False)

   (e): The subset of $\mathbb{R}^\omega_{box}$ defined by:
   $$\mathbb{R}^\infty = \{(x_1, x_2, .., x_n, ...) \in \mathbb{R}^\omega_{box} : x_i = 0 \text{ for all but finitely many } i \in \mathbb{N}\}$$
   is a closed subset of $\mathbb{R}^\omega_{box}$. (True/False)

   (f): Let X and Y be two topological spaces such that X is homeomorphic to a subspace of Y and Y is homeomorphic to a subspace of X. Then X is homeomorphic to Y. (True/False)

In the remaining three questions below, provide arguments to justify your answer. Anything proved in class can be used by citing the result, without reproducing the proof. The results of exercises however, have to be proved in full.

2. Let $f : \mathbb{R} \to S_\Omega$ be a continuous map. Prove that f is a constant map. (12 mks)

3. Prove that the ordered square $I_o^2 = [0,1] \times [0,1]$ (with dictionary order topology) is first countable. (12 mks)

4. Let $S^1 = \{z \in \mathbb{C} :\mid z \mid= 1\}$ and $D^2 = \{z \in \mathbb{C} :\mid z \mid\leq 1\}$. Prove that the map $f : S^1 \to D^2 \times S^1$ given by $f(z) = (z, z)$ is not homotopic to a constant map. (12 mks)